Conspiracy in senior school mathematics

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Research across five countries has identified inability to pay attention to mathematical detail – the discipline of noticing – is an issue in senior secondary school mathematics teachers. The test and questionnaire completed by an Australian cohort further identifies a reluctance to employ non-routine questions in assessments, with teachers concerned about damaging the trust relationship they enjoy with their students. As teachers fail to demonstrate strong ability in non-routine written test questions themselves, this paper questions whether there exists a 'conspiracy' between teachers and their students to avoid scrutiny of conceptual understanding.

Introduction

In mathematics education, systematic reflective thinking may involve interrogation of practice both inwardly and outwardly (Mason, 2002). Inward reflection may be practical, a consideration of such matters as classroom management and appropriate delivery of the curriculum. This may lead to a metacognitive reflection, as the practitioner searches for the causes of their confidence in the techniques they are applying and a search for their personal assumptions and abilities. At a higher stage again, Mason (2002, p. 17) identifies "social-reflection", a more outward-directed critique of the values which impose upon the teaching situation.

A key concept in reflective thinking is intentional noticing. Mason (2002) relates an anecdote about pianist Artur Rubinstein deliberately choosing to not use a certain finger in a concert, just to be more aware of his playing. In teaching practice, self-noticing can be practiced in respect of gesture, how conversation is initiated or terminated, or of the things the practitioner chooses to note down in writing. Underlying Mason's approach is the idea that, through conscious practise, noticing and ultimately teaching performance can be improved.

This study seeks to apply Mason's (2002) concepts of intentional noticing to assessment of mathematics. Incorrect responses to mathematical questions can result from overlooking aspects of the problem. For example, in calculus a local maximum may be obtained by differentiation when, for the defined domain, the global maximum may be greater and optimal. Overlooking discontinuities in functions is another difficulty in the mathematical performance of school students. In the opaque language of Examiners Reports, a statement such as "Standard questions involving calculus, logarithms and the exponential function were well attempted" (School Curriculum and Standards Authority, 2018, p. 1) can be taken to indicate that the unmentioned non-standard questions were not subject to sufficiently close attention.

This paper recounts research by Klymchuk (2014) on the phenomenon of lack of attention in mathematics students and in their teachers. It describes replication in an Australian setting of Klymchuk's study and presents an account of the self-knowledge of the Australian teachers involved. Klymchuk's conclusion is that "Solving non-routine, non-standard questions would better prepare students for the real world. Enhancing their

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own and their students' discipline of noticing by paying attention to details can also be a useful addition to teachers' professional development." (p. 69). This paper explores whether these sentiments apply in an Australian situation and reports the self-interpretation of the actors involved. It explores whether the intentional noticing of Mason (2002) may be a means of improving the mathematical performance of teachers and students in Australia.

Theoretical Framework and Literature Review

Intentional noticing appears under several names in educational research. Schön (1987) favoured the term "reflection", but partitioned it into such categories as "reflection-in-action". Mason (2002) offers further varieties of reflection, examining intention and motivation. Noticing is treated as a study of student misconceptions in works including Ryan and Williams (2007). More recently, mindfulness in mathematics is a theme of Boaler (2016), developing work by Dweck (2007).

Duit, Treagust and Mansfield (1996) note that from a constructivist perspective there is a symmetrical relationship between teacher and school students, both being partners in a communication and trying to obtain an idea of the understanding of the other. With this in mind, it is important to examine the self-perceptions of the teachers, not just the performance of the students, and also the joint behaviour and understandings of teachers and students.

To move from habitual or mechanical patterns in teaching practice, Mason (2002) recommends teachers undertake a series of exercises to develop increased sensitivity. These include mirroring gestures, listing key words for a lesson and introspection about achievement of desired intentions in interactions with students. This same intentional improvement in performance is possible in mathematics, claim Meyer, Falkner, Sooriamurthi and Michalewicz (2014). Their solution is exposure to carefully-selected mathematical puzzles where close attention is needed.

> Puzzle-based Learning is rapidly becoming a bigger and bigger part of the curriculum as there is no guarantee that a traditional education will provide students with enough practise and experience to develop problem-solving skills. The rapidly changing face of employment and technology means that the problems that we train people to solve today are probably not the problems they will be solving in ten years. When our current education system tends to favour highly focused learning of rigid approaches to predictable problem sets, there is no guarantee that our students will be flexible enough and resilient enough to cope with open-ended problems with no guaranteed solution. (Meyer et al., 2014, p. 4)

Research originally undertaken by Klymchuk (2014) in New Zealand was then was replicated by his associates in three other countries: Hong Kong, Germany and Ukraine. Klymchuk describes the New Zealand group as "experienced upper secondary school mathematics teachers"; the Hong Kong group as "secondary school mathematics teachers"; the German group as "experienced school mathematics teachers"; and the Ukraine group as "[Tertiary] Year 3-4 mathematics students training to become secondary school mathematics teachers with the majority having had teaching experience as part of their training" (p. 64). Klymchuk distinguished two clusters: in the New Zealand and German groups "roughly half of the participants were disappointed and embarrassed while the other half were more positive and saw the opportunity for improvements", whereas the Hong Kong and Ukraine groups "the vast majority were very disappointed and uncomfortable". Klymchuk indicated "The difference between the two clusters might be due to culture" (p. 67).

Klymchuk (2014) designed a test and questionnaire to explore the role of reflective thinking in mathematics assessment and to search for the underlying reasons for incorrect answers. The test questions – as given in Appendix A, with solutions – are described by Klymchuk as "provocative" in the sense that, although they appear routine, each of the seven questions contains a non-routine 'catch' which relies on conditions and constraints within the mathematics. For example, the third question asks respondents to "Solve the equation $ln(x^2 + 17x - 18) - ln(x^2 + 5x - 6) = 0$". The initial impression may be that this question will succumb to standard mathematical manipulations, however the domain of the logarithm function is restricted to numbers greater than zero, therefore no real value of x satisfies the equation.

Mason's (2002) "discipline of noticing" involves paying attention to such detail as conditions, constraints, locality, properties and relationships. It was expected that the various 'catches' in Klymchuk's test would result in some participants obtaining incorrect answers, even though some countries were represented by experienced mathematics teachers. In practice, Klymchuk (2014, p. 63) reported that "The results of the test were startling – the vast majority of the participants gave incorrect answers to most questions in the test."

Methodology

In order to replicate the research undertaken by Klymchuk (2014) in an Australian setting, an account of the exact conditions under which the test and questionnaire were administered was obtained. The English language version of the test and questionnaire were adopted verbatim. The Australian survey participants were delegates at a November 2017 conference of secondary mathematics teachers, thereby ensuring all were practicing secondary mathematics teachers. The participants were invited to take part in a conference session entitled "An experiment which may change your teaching practice" with this session abstract: "We will do a test which contains some routine questions and some trick ones. Some are calculus questions. We will discuss the answers and consider the implications for our teaching." The expectation was that the mention of calculus would dissuade attendance by mathematics teachers whose experience was only of junior secondary classes.

Sixty teachers attended, completing the 15-minute test and a short questionnaire about their test results after the solutions had been discussed. In order to maintain conformity with the research in other countries, the wording of the session abstract, test questions, solutions and questionnaire prompts were used verbatim as reported in the original study (Klymchuk, 2014), with the same procedure and timing. The seven-question test was distributed as a write-on paper while a 15-minute countdown timer was visible on a projector screen. Although participants were seated adjacent to each other around large tables, no discussion was permitted and no such cooperation was observed.

In Western Australia three of the senior school mathematics courses have summative examinations to which school-based assessments are moderated: Mathematics Specialist, Mathematics Methods and Mathematics Applications. These are collectively titled the ATAR [Australian Tertiary Admission Rank] courses. The test questions discussed in this paper are based on content from ATAR mathematics courses. As evident from their test responses, in four cases the teachers demonstrated no acquaintance with the content of the ATAR courses, only with algebra. Although they were attending a conference for secondary mathematics teachers, these four may teach only junior classes and / or may be teaching out-of-field without a tertiary mathematics background.

Mathematics Test Results

The percentage of correct answers for each question, calculated against the number of completed test papers, is presented in Table 1. The percentages are a comparison of the number of correct solutions for the question, marked either fully correct or incorrect, against the country cohort size n. Klymchuk (2014) is the source of the non-Australian data.

Table 1

Percentage of Correct Solutions in Non-Routine Senior Secondary Mathematics Questions

		Question number						
Country	*n*	1	2	3	4	5	6	7
Australia	57	2	54	33	4	28	9	28
Germany	10	0	60	30	NA	20	0	0
Hong Kong	26	23	12	27	19	12	15	12
New Zealand	14	0	7	21	7	0	8	0
Ukraine	8	26	0	19	31	15	12	0

In the Australian study, 60 people were present to undertake the test, but only 57 test papers were received. Of these, there were 17 teachers who failed to obtain any correct solutions. It was evident from blank responses to questions that many of the Australian teachers had little idea of how to approach the questions at all, and the various ‘catches’ proved difficult even for those who were able to make a start. As in the research in other countries, no information was gathered directly from participants about their extent of training in mathematics nor length of experience as mathematics teachers.

Several misdirecting factors may have contributed to low scores. Despite indication in the session abstract that “some” trick questions would be included, in fact what Klymchuk (2014) refers to as ‘catches’ existed in every one of the seven questions. Some indicative questions may be seen in Appendix A. In Q1 the phrasing “height dropped on the hypotenuse” may have been unfamiliar to some participants. Q3 and Q5 demanded solving where there is no solution. Q4 directs participants to “prove the identity” of an equation which is not an identity at all. Q6 and Q7 ask participants to “find” items which actually do not exist. Such directly-phrased instructions may have been taken by the Australian cohort as indication that the task is feasible. Also, a typographical error (the symbol for “equals zero” was omitted) existed in Q5 until corrected verbally during the test.

Questionnaire Results

When the test papers were collected in, solutions were distributed and discussed and a follow-up questionnaire was distributed. 51 completed questionnaires were received. The questionnaire paper contained just three prompts: “What are your feelings after you have learnt about the correct solutions to the test questions?”, “What are the reasons for not solving all test questions correctly?” and “Would you make any changes in your teaching practice after doing the mini-test. If so – what changes? If not – why?". Fifty-one completed questionnaires were received. They are reported here without regard to the prompt as the written responses often were not specific to a single prompt. The themes which emerged are frustration, self-awareness, a trust relationship between teachers and students, and a perception that procedural understanding is prioritised by the examination system.

The responses were phrased politely but they do give an idea of the degree of exasperation and concern felt by some of the teachers: "Annoyed at being tricked by some obvious things", "I've been knocked down a peg or two. While doing the questions I felt confident in my ability and did spot a couple of the conundrums but it turns out that most of them got past me", "We were stitched up followed by great learning opportunity", "Ashamed because I did not get the correct answers", "I think this is an 'immoral' test. I don't believe you should ask someone to prove something is true when it isn't", "Make sure questions work and don't be nasty", "I didn't pay attention to the fundamental assumptions for a concept!", "I was too busy doing the mechanics of the questions and didn't take time to look carefully at all parts of the questions presented to see if they truly exist", "I would like to re-sit the test so that I can check if I have learnt from errors", "Tricked", "A little foolish, surprised", "Frustration, disappointment", "Not paying attention", "Excitement, satisfaction, curiosity". Responses such as "Not paying attention" reflect Klymchuck's (2014) subtitle "drawing attention to a lack of attention", but other responses do not. Many Australian respondents did not identify in themselves a lack of attention to mathematical detail, but rather they identified improper questions – questions which did not accord with the expectations of the respondents because they incorporated difficult and unexpected elements. These are elements which required noticing of detail. The Australian cohort are best associated with the New Zealand and German cluster, where there is divergence of response, rather than the Hong Kong and Ukraine cluster where subjects focused strongly on deficiencies in their own performance. This may indicate "social-reflection" values as identified by Mason (2002) or "culture" as identified by Klymchuk (2014).

In the Australian cohort, a dozen respondents (24%) in their questionnaire comments noted their own lack of knowledge, often coupled with indication of the reasons: "Lack of practice – have not studied / applied maths at this level for 12 years", "I had forgotten some concepts which I haven't used in a long time, as I don't teach methods & specialist at the moment", "I am not maths trained", "Didn't know the topic to that depth", "I failed to see each question had underlying principles or assumptions that I did not remember".

The word "trust" featured in six responses (12%). It was used to indicate that teaching and assessment is a bond between teachers and students. "Students should be able to trust the questions", "When students are doing a test they shouldn't be looking for a trick all the time. It is a matter of trust.", "[I have] basic trust test questions are mostly correct!!!", "I ignored whether the solutions were possible, i.e. I trusted the questions were "real" and [solutions] existed", "Trust. Didn't check all solutions. I did pick up some issues and became suspicious", "I didn't even think to check whether or not the triangle was possible as there was a "trust" that the triangle is real if the examiner is asking for an area. ... Q5 I showed that there are no solutions but crossed out my working, again having a "trust" in the examiner's questions". In a similar vein, other responses included: "In my teaching practice my students are given only problems which are possible to solve. They follow the script.", "I would hope that the majority of my questions are already checked and are workable as they are." "At school we don't tend to pose impossible questions", "I feel a teacher giving these questions would be unfair”. At the conclusion of the session, one teacher approached the investigator to share that such ‘trick’ questions are inappropriate for school students because they “destroy the trust of my students”. She encourages young teachers to work the examination questions themselves so that no impossible questions are presented. Another teacher privately intimated that he always tells the students there are no ‘trick’ questions in the test, otherwise the students waste too much time looking for them.

One factor which figured prominently in the responses was that the teachers saw preparation for examinations as the pre-eminent priority: “Exams always have questions which make sense, so why teach them beyond the process?”, “I am worried that in an assessment they will become absorbed by looking for the trick and waste precious time, as the assessments they do, do not have trick questions.”, “We are trying to get them to be successful in their WACE [West Australian Certificate of Education] exams after all.” Twenty-five respondents (49%) used the near-equivalent words “process”, “mechanics”, “methods”, “procedure”, “algorithms”, “routines” and “strategies”. These terms were used to indicate that the test-taking activity was viewed as a predictable pathway. Gaining familiarity with this pathway was preparation for examinations. But, as one teacher noted, “It is very easy to follow a rule / algorithm / formula, but unless you have the understanding ‘why’ you cannot see when there may be no solution”.

A need for students to “question the questions” was recommended by 27 respondents (53%) who indicated that they will make use of such ‘trick’ questions, but only with their most able students for two respondents. Conversely, 23 respondents (45%) indicated it is contingent on teachers to ensure all questions have feasible solutions. These respondents did not see value in ‘trick’ questions which call for greater scrutiny of the conceptual underpinnings of the content, or they felt such considerations should not be included in assessments.

Discussion

The key observation about the overall results in Table 1 is that many teachers were unable to answer the questions. In many of the test questions it was not the ‘catch’ which caused Australian respondents to fail the question, it was lack of application of the routine methods of solution. In Question 5, for example, 28 of the 57 teachers did not employ the Intermediate Value Theorem nor any other productive means of attack. It was not that the function contains a discontinuity which was problematic: the teachers were unaware of basic function analysis technique in the first place.

Subtle linguistic cues may influence teachers in different cultures and operating in different languages. For some people it may be discourteous to respond “this cannot be done” to a question posed in a university-warranted test. Australian teachers may have little experience in mathematical questions which have no solution – or a multiplicity of solutions – whereas the format may not be unconventional in other countries.

If senior secondary mathematics assessments contain few questions which explore full understanding, the questions presented to “trusting” students must be routine, the content pre-negotiated and expected. Students who are not exposed to the risk of losing their self-confidence are therefore not being challenged in assessments to demonstrate more than procedural competence. This suggests there may be a conspiracy between teachers and their students to avoid coverage of true conceptual understanding in senior school mathematics tests and examinations.

The idea of ‘conspiracy’ in relation to the assessment of mathematics in Australia has appeared before. The architect of the current mathematics curriculum, Professor Peter Sullivan, writes on the use of open-ended tasks:

> One of the major constraints that teachers experience when utilising such tasks is that many students avoid risk taking and do not persist with the challenges that are required in order to complete the task. And teachers are sometimes complicit in this avoidance strategy. Desforges and Cockburn (1987), for example, reported on a detailed study of primary classrooms in the United Kingdom and found that students and teachers conspired with each other to reduce the level of risk for the students. (Sullivan, 2010, p. 38)

Although Sullivan (2010) discusses primary students, his point is that teachers can be complicit in an avoidance strategy.

The local mathematics curriculum (School Curriculum and Standards Authority [SCSA], n. d.,) states in the Rationale of each of the ATAR curricula that “For all content areas of the ... course, the proficiency strands of the Year 7–10 curriculum continue to be applicable and should be inherent in students’ learning of the course. These strands are Understanding, Fluency, Problem-solving and Reasoning ...”. The curriculum requirements include that students “interpret mathematical information and ascertain the reasonableness of their solutions to problems”, a phrase repeated in the Learning Outcomes section in every Unit of the ATAR curricula. Senior secondary students therefore should encounter content such as boundary conditions, discontinuities and limitations to definitions. Provision exists within senior secondary mathematics school-based assessment to provide investigation of such topics as the limitations on and counter-intuitive properties of functions. For all three ATAR courses the curriculum (SCSA, n. d.) notes of the school-based Investigation tasks “This assessment type provides for the assessment of general inquiry skills, course-related knowledge and skills, and modelling skills.” If teachers have only a procedural understanding of mathematics themselves, the school-based assessments they provide may just mimic examinations and fail to include these wider skills.

The question arises whether the ATAR examinations themselves could do more to allow “paying attention” as Klymchuk (2014) phrases it. In the 2017 Mathematics Methods examination (School Curriculum and Standards Authority, 2017) Q9 requires calculation of a linear model but later allows students to reject the model on the basis of patterns in residuals; Q10 asks students to select appropriate “equation(s)” where four are offered and two of them are appropriate; Q12 has marks for students declaring a given interpretation is incorrect because “cause is not established”; Q14 “find any point of inflection” has no point of inflection because the function is undefined at the only value of x where the second derived function is equal to zero. Unlike Klymchuk’s questions, the examination does not direct students to find solutions which do not exist, and there is little of the “noticing” advocated by Mason (2002). An in-depth study of Australia’s senior secondary mathematics teaching and examinations is beyond the scope of this paper, but the examination cited provides very mild support, if any, that the questions allow students to employ Mason’s “noticing”.

Conclusions

The Australian teachers had difficulty recognising ‘trick’ questions, and many expressed a disinclination to employ such questions in their own teaching. This study reveals that a significant number of teachers are unable to tackle hard mathematics questions on a routine basis in the first place. Avoidance of difficult material is not in the best interests of students and defensive responses by teachers may indicate a need for increased provision of in-service professional learning. Senior secondary teacher preparation courses mandate tertiary-level mathematics units. But tertiary mathematics does not constitute the material taught in secondary schools. Passing a tertiary mathematics unit does not ensure deep understanding of concepts in secondary school mathematics, an area in which secondary mathematics teachers may be wanting.

Australian teachers greatly value the trust relationship they enjoy with their students. However, this seems to deter some teachers from utilising questions which call on students to demonstrate deep conceptual understanding and confident exhibition of self-belief. Greater use of puzzle-based learning and intentional noticing may well prove advantageous in Australian schools.

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Appendix A

Indicative "provocative" questions posed by Klymchuk (personal communication, March 13, 2017).

2. Find the domain of the function $y = f(g(x))$ if $f(x) = x^2 + 1, \quad g(x) = \sqrt{x-2}$

3. Solve the equation $ln(x^2 + 17x - 18) - ln(x^2 + 5x - 6) = 0$

5. Show that the equation $\frac{x^2+\sqrt{x}+1}{x-1} = 0$ has a solution on the interval $[0, 2]$

6. Find the derivative of the function $y = ln(2sin(3x) - 4)$